Chapter 5
Normal Probability Distributions

Section 5-1 – Introduction to Normal Distributions and the Standard Normal Distribution

A. The normal distribution is the most important of the continuous probability distributions.
   1. Definition: A normal distribution is a ________________ for a random variable x.
      a. The graph of a normal distribution is called a ____________.
   2. A normal distribution has the following properties:
      a. The ___________, ____________, and ___________ are equal (or VERY close to equal).
      b. The normal curve is ________________ and is ________________ about the ____________.
      c. The ________________ under the normal curve is equal to ________.
      d. The normal curve ________________, but never touches, the ____________ as it gets ________ away from the ____________.
      e. The graph curves ________ within ________ standard deviation of the mean, and it curves ________ outside of ________ standard deviation from the mean.
         1) The points where the curve changes from curving upward to curving downward are called __________________________.

B. We know that a discrete probability can be graphed with a histogram (although we didn’t emphasize this in Chapter 4).
   1. For a continuous probability distribution, you can use a ________________ (______).
      a. A probability density function has two requirements:
         1) The ________________ under the ________________ has to equal ________.
         2) The function can never be ________________.
   2. A normal distribution can have ANY ____________ and ANY POSITIVE ____________ ____________.
      a. These two parameters completely determine the ____________ of the normal curve.
         1) The mean gives the ____________ of ________________.
         2) The standard deviation describes how ____________ (or bunched up) the data is.

C. The Standard Normal Curve
   1. There are ________________ normal distributions, because there are ________________ ____________ possible combinations of ________________ and ________________ ____________.
      a. The ________________ ____________ has a mean of ________ and a standard deviation of ________.
         1) The ________________ of ________________ of the standard normal distribution corresponds to ________________.
            a) Remember that z-scores are ________________ of ________________ that indicate the number of ________________ values lie away from the mean.
               1. \( z = \frac{x - \mu}{\sigma} \)
   2. The standard normal distribution has the following properties:
      a. The ________________ area under the curve is close to 0 for z-scores close to ________.
      b. The ________________ area increases as the z-scores ________________.
c. The area for \( z = 0 \) is ________________.
d. The area is close to 1 for \( z \)-scores close to ____________.

3. To find the corresponding area under the curve for any given (or calculated) \( z \)-score, there are two main methods.
   a. The easiest, and the one I suggest, is to use the TI-84 calculator.
      1) \( 2^{nd} \) VARS normalcdf (lower boundary, upper boundary)
         a) If you want the area to the left of a \( z \)-score, use -1E99 as your lower boundary and the \( z \)-score you are interested in as your upper boundary.
         b) If you want the area to the right of a \( z \)-score, use the \( z \)-score you are interested in as your lower boundary and use 1E99 as your upper boundary.
         c) If you want the area between two \( z \)-scores, use them both (smaller one as lower, larger one as upper).
   b. In terms of \( z \)-scores, this means that a \( z \)-score of less than ______ or greater than ______ means an unusual event.
      1) A \( z \)-score of less than ____ or greater than _____ means a very unusual event (Outlier).

4. Remember that in Section 2.4 we learned from the Empirical Rule that values lying more than ______ _______ _______ from the mean are considered to be unusual.
   a. We also learned that values lying more than ______ _______ _______ from the mean are very unusual.
   b. In terms of \( z \)-scores, this means that a \( z \)-score of less than ____ or greater than ____ means an unusual event.
      1) A \( z \)-score of less than ____ or greater than ____ means a very unusual event (Outlier).

The examples on the PowerPoint are all except 1) in your book; I am not printing them here to save paper. Either follow along in the book or take notes on your own paper, or both.

Page 250, #42 requires us to go all the way back to the frequency distribution table that we did in Chapter 2. Refer to your notes from Section 2-1 for that. I am including a blank table here for you to fill in as we go over this problem.

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Section 5-2 – Normal Distributions: Finding Probabilities
A. On the STANDARD normal curve, the mean is always 0 and the standard deviation is always 1, and we always use \( z \)-scores.
   1. There are an infinite number of possible normal curves, each with its own mean and standard deviation.
   2. To find the probability of any particular \( x \) value in one of these other normal distributions, we will use the same distribution on the calculator, simply changing the mean and standard deviation to match the data.

The examples on the PowerPoint are all in your book; I am not printing them here to save paper. Either follow along in the book or take notes on your own paper, or both.
Section 5-3 – Normal Distributions: Finding Values

A. We have learned how to calculate the ______________ given an ______________ or a ______________. In this lesson, we will explore how to find an ______________ or ______________ when given the ______________ (cumulative area under the curve).

1. The ______________ area under the curve is a ______________ ______________ of the ___________; as the ______________ goes up, so does the ______________ ______________.
   a. Because this is a one to one function, it also has an ______________ function.
      1) Lucky for us, the calculator has an operation to find the inverse of the cumulative area.
         a) 2nd VARS (______________) = ______________.
         b) 2nd VARS (______________, ______________, ______________) = ______________.

B. The key here is going to be using the ______________ area under the ______________ to find the z-score or x value that we are looking for.

1. Practice will make this a LOT easier.
   a. As a general rule, if we want the area ______________ a given percentile, we use the ______________ ______________.
   b. If we want the area ______________ a given percentile, we ______________ the ____________ and use the ______________ ______________ from ______ and use the answer.

Section 5-4 – Sampling Distributions and the Central Limit Theorem

A. A ______________ distribution is the ______________ distribution of a ______________ statistic that is formed when samples of size n are ______________ taken from a ______________.

1. If the sample ______________ is the sample ______________, then the distribution is the ______________ distribution of the sample means.
   a. Every sample statistic has a sampling distribution.

2. Remember that sample means can ______________ from one another and can also vary from the ______________ mean.
   a. This type of variation is to be ______________ and is called ______________.

3. Properties of sampling distributions of sample means:
   a. The ______________ of the sample means is ______________ to the population ______________.
   b. The ______________ of the sample means is equal to the population ______________ divided by the ______________ of n.
      1) The standard deviation of the sampling distribution of the sample means is called the ______________.

B. The Central Limit Theorem

1. The Central Limit Theorem forms the ______________ for the ______________ branch of statistics.
   a. It describes the ______________ between the sampling distribution of ______________ means and the ______________ that the ______________ are taken from.
   b. It is an important tool that provides the information you’ll need to use sample statistics to make ______________ about a ______________ mean.

2. The Central Limit Theorem says:
   a. If samples of size n, where n ≥ ________, are drawn from any population with a mean μ and a standard deviation σ, then the sampling distribution of sample means ______________ a ______________ distribution.
1) The greater the sample size (the larger number \( n \) is), the __________ the approximation.

b. If the __________ itself is __________ distributed, the sampling distribution of sample means is __________ distributed for __________ sample size \( n \).

3. Whether the original population distribution is normal or not, the sampling distribution of sample means has a mean __________ the population mean.

a. In real life words, this means that if we take the average of all of the means, from all of the samples that are done on one population, the mean of those averages will equal the mean of the population.

4. The sampling distribution of sample means has a __________ equal to 1/\( n \) times the __________ of the __________.

a. The standard deviation of sample means will be __________ than the standard deviation of the population.

5. The sampling distribution of sample means has a standard deviation __________ to the __________ standard deviation divided by the square root of __________.

a. The distribution of sample means has the same center as the population, but it is not as __________.

1) The __________ \( n \) (the sample size) gets, the __________ the standard deviation will get.

a) The more times we take a sample of the same population, the more tightly grouped the results will be.

b. The standard deviation of the sampling distribution of the sample means, \( \sigma_x \), is also called the __________

C. Probability and the Central Limit Theorem

1. Using what we've learned in Section 5-2, and what we've been told here in Section 5-4, we can find the probability that a sample mean will fall in a given interval of the sampling distribution.

a. To find a \( z \)-score of a random variable \( x \), we took the value minus the mean and divided by the standard deviation.

b. To convert the sample mean to a \( z \)-score, we alter that slightly.

1) Instead of dividing by the standard deviation, we divide by the sample error.

a) Remember, this is the standard deviation of the population divided by the square root of \( n \) (the sample size). 
\[
z = \frac{x - \mu}{\sigma/\sqrt{n}}
\]

Section 5-5 – Normal Approximations to Binomial Distributions

A. Properties of a Normal Approximation to a Binomial Distribution

1. If \( np \geq _____ \), and \( nq \geq _____ \), then the binomial random variable \( x \) is approximately normally distributed, with a mean that equals _____ and a standard deviation that equals ________.

a. Again, if \( np \geq _____ \), and \( nq \geq _____ \), then \( \mu = _____ \) and \( \sigma = _____ \).

b. We need to remember from Section 4-2 what the properties of a binomial experiment are:

1) \( n \) ________________ trials (we know before we start how many trials there are going to be).

2) Only ________ possible outcomes (success or failure).

3) Probability of success is ____.

4) Probability of failure is \( 1 - p \), which we call ____.

5) \( p \) is ________________ for each trial (the trials have nothing to do with each other).

2. Correction for Continuity

a. Binomial distributions only work for ________________ data points.

1) When we want to calculate the **exact** binomial probabilities, we can find the probability of each value of \( x \) occurring and add them together. We did this in Chapter 4.
b. To use a normal distribution to approximate a binomial probability, you need to move _____ unit to each side of the __________ to include all possible x-values in the interval.

1) This is called making a ______________.
   a) We subtract _____ units from the lowest value and add _____ units to the highest value.

3. There is a good review chart with this information displayed on page 288 of your text book.
   a. The steps to using the Normal Distribution to Approximate Binomial Probabilities are:
      1) Verify that the binomial distribution applies.
         a) Specify n, p, and q.
      2) Determine if you can use the normal distribution to approximate x, the binomial variable.
         a) Are np and nq both greater than or equal to 5?
      3) Find the mean and standard deviation for the distribution.
         a) \( \mu = np \) and \( \sigma = \sqrt{npq} \).
      4) Apply the approximate continuity correction. Shade the corresponding area under the normal curve.
         a) Subtract .5 unit from lowest value, add .5 unit to highest value.
      5) Find the corresponding z-score(s).
         a) \( z = \frac{x - \mu}{\sigma} \)
      6) Find the probability.
         a) Use the calculator.